Covered Clauses Are Not Propagation Redundant

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Overview

- SAT success by reasoning about **redundancy**
  - Non-equivalence-preserving solving techniques
  - Strong propositional proof systems
- Revisit **covered clauses** [5]
  - Generalization of blocked clauses [14]
  - Used for clause elimination, preprocessing
- Consider CCs in more recent context of redundancy
  - Redundancy characterized via **witnesses**
  - Proof systems like SPR and PR [7]

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- Consider CCs in more recent context of redundancy
  - Redundancy characterized via **witnesses**
  - Proof systems like SPR and PR [7]

- Present an algorithm to identify CCs
- Show CCs are not generalized by PR
- Prove witnesses for CCs are hard to compute
- Deciding clause redundancy is co-DP-complete

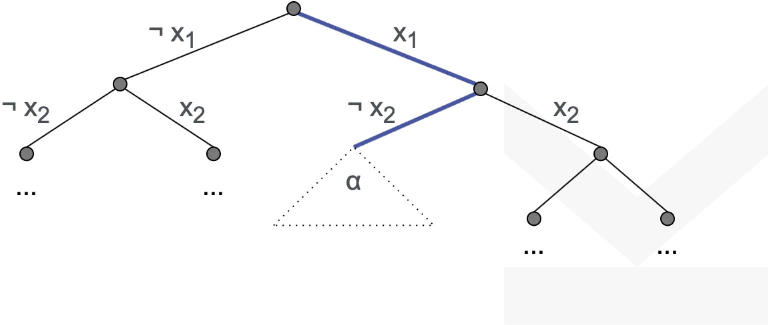
Background

Redundancy [13]

Clause C is **redundant** w.r.t. formula F if:

F and $F \wedge C$ are **equisatisfiable**

- Suppose $(\neg x_1 \vee x_2)$ is redundant w.r.t. F



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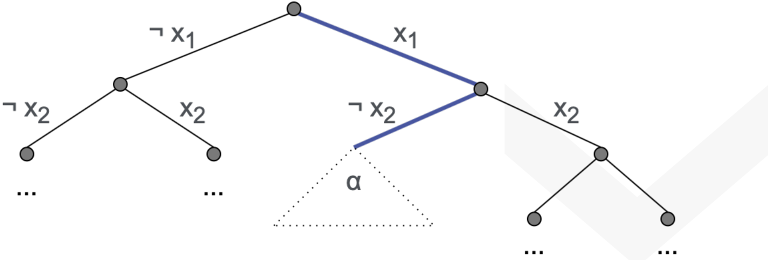

- If α includes solutions, there are solutions elsewhere too

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(2) Proof systems: add $C \notin F$ such that $P(F, C)$, eventually add $\perp$

- (Resolution), DRAT [16], SPR, **PR** (Propagation Redundancy)
- PR has simple, short proofs of **pigeonhole formulas** [7] (shortest res. proofs are exponential in size [3])

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- (Resolution), DRAT [16], SPR, **PR** (Propagation Redundancy)
- PR has simple, short proofs of **pigeonhole formulas** [7] (shortest res. proofs are exponential in size [3])

* Strong even without extension (new variables), deletion
* Seem to be somewhat "automatizable"

Background

Witnesses [7, 9]

HKB [7]: C is redundant w.r.t. $F \iff$ for $\alpha = \neg C$, there exists ω:

$$\begin{array}{lrcl} (1) & \boldsymbol{\omega} & \models & \boldsymbol{C} \\ (2) & \boldsymbol{F}|_\alpha & \models & \boldsymbol{F}|_\omega \end{array}$$

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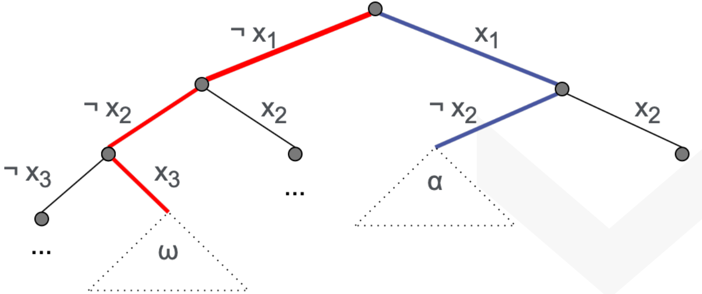

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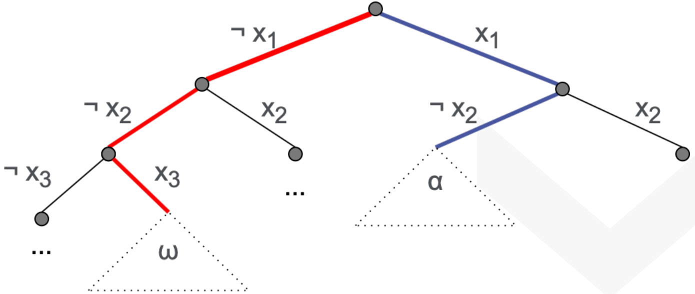

- $C = (\neg x_1 \vee x_2)$
- $\omega = \neg x_1, \neg x_2, x_3$ is a witness if $F|_{x_1,\neg x_2} \models F|_{\neg x_1,\neg x_2,x_3}$

Background

Witnesses [7, 9]

Clause elimination: witnesses needed for **reconstruction** [4]

- Remove redundant clauses C_1, C_2, ..., C_N from F
- Solution τ for $F \setminus \{C_1, \dots, C_N\}$ may not satisfy F
- Record witness ω_i for each C_i, apply **reconstruction function**

$$\mathcal{R}_\epsilon(\tau) = \tau \qquad \mathcal{R}_{\sigma\cdot(\omega:C)}(\tau) = \begin{cases} \mathcal{R}_\sigma(\tau) & \text{if } \tau \vDash C \\ \mathcal{R}_\sigma(\tau \circ \omega) & \text{otherwise} \end{cases}$$

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Witnesses crucial for PR proofs as well

C is PR w.r.t. F if $F|_\alpha \vdash_1 F|_\omega$ for some $\omega \vDash C$

- Deciding if C is PR w.r.t. F is NP-complete [10]
- Must record witness for each added C for polynomial proof check

Background

Witnesses [7, 9]

Property	Witness ω	Implication check
Subsumption	any*	$F\|_\alpha \ni \bot$
RUP [2]	any*	$F\|_\alpha \vdash_1 \bot$
Blocking	$\alpha \circ \ell$ for some $\ell \in C$	$F\|_\alpha \supseteq F\|_\omega$
RAT	$\alpha \circ \ell$ for some $\ell \in C$	$F\|_\alpha \vdash_1 F\|_\omega$
Set-blocking [13]	$\alpha \circ L$ for some $L \subseteq C$	$F\|_\alpha \supseteq F\|_\omega$
SPR	$\alpha \circ L$ for some $L \subseteq C$	$F\|_\alpha \vdash_1 F\|_\omega$
Global-blocking [12]	$\alpha \circ L$ for some $L \cap C \neq \emptyset$	$F\|_\alpha \supseteq F\|_\omega$
PR	any	$F\|_\alpha \vdash_1 F\|_\omega$
R	any	$F\|_\alpha \vDash F\|_\omega$

- $F|_\alpha \vdash_1 \bot$ means unit propagation on $F|_\alpha$ produces $\bot$
- $F|_\alpha \vdash_1 F|_\omega$ means $F|_\alpha \wedge \neg D \vdash_1 \bot$ for every $D \in F|_\omega$

Covered Clauses

Definition [5, 6]

- Consider the resolvents of $C = (a \vee b)$ on a:

$$\boldsymbol{C \otimes_a (D \vee \neg a)} \text{ for all } (D \vee \neg a) \in F$$

All resolvents tautological	$\Rightarrow$	a blocks C
All non-taut. resolvents include x	$\Rightarrow$	a **covers** x

- Extend C by adding covered literals: $C_{\text{ext}} = (a \vee b \vee x)$

$$(F \wedge C_{\text{ext}})|_{\neg a, \neg b} \ \vdash_1 \ (F \wedge C_{\text{ext}})|_{a, \neg b}$$

- C is redundant w.r.t. $F \wedge C_{\text{ext}}$, with witness $\boldsymbol{\omega = a, \neg b}$
- Iteratively add covered literals to C_{ext}

C is **covered** if some extension C_{ext} is blocked, or subsumed.

Covered Clauses

Example

(1) a covers x	$(a \vee b)$
	$\Downarrow$
(2) b covers y	$(a \vee b \vee x)$
	$\Downarrow$
(3) C_{ext} blocked by x	$(a \vee b \vee x \vee y)$
	$\Downarrow$
(4) $(a \vee b)$ is covered	$\top$

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(2) b covers y	$(a \vee b \vee x)$	$\omega_2 = \neg a, b, \neg x$
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- Reconstruction for CC-Elimination: use the sequence of witnesses

$$\begin{array}{ll} (\omega_3 : & a \vee b \vee x \vee y) \\ (\omega_2 : & a \vee b \vee x) \\ (\omega_1 : & a \vee b) \end{array}$$

Identifying Covered Clauses

- Actually identify asymmetric covered clauses [6]
- Can extend C by both covered and **asymmetric** literals ℓ:

$$(D \vee \neg\ell) \in F \text{ for some } D \subseteq C$$

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- Can extend C by both covered and **asymmetric** literals ℓ:

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- Adding asymmetric literals is equivalence-preserving
- ACC is strictly more general than CC

Identifying Covered Clauses

$\mathsf{ACC}(F, C)$

$\sigma := \varepsilon$

$E := C$

$\alpha := \neg C$

repeat

  if $\bot \in F|_\alpha$ **then return** $(\textbf{true}, \sigma)$

  if there are unit clauses in $F|_\alpha$ **then**

    $\alpha := \alpha \cup \{u\}$ for each unit u

  else

    for each $l \in E$

      $G := \{D|_\alpha \mid (D \vee \neg l) \in F \text{ and } D|_\alpha \neq \top\}$

      if $G = \emptyset$ **then return** $\big(\textbf{true}, \sigma \cdot (\neg E_l : E)\big)$

      $\Phi := \bigcap G$

      if $\Phi \neq \emptyset$ **then**

        $\sigma := \sigma \cdot (\neg E_l : E)$

        $E := E \cup \Phi$

        $\alpha := \alpha \cup \neg \Phi$

until no updates to α

return $(\textbf{false}, \varepsilon)$

Identifying Covered Clauses

ACC(F, C)

$\sigma := \varepsilon$ — witness sequence

$E := C$ — C with covered literals

$\alpha := \neg C$ — C with all added literals (negated)

repeat

  if $\perp \in F|_\alpha$ **then return** (**true**, σ) — Subsumption Check

  if there are unit clauses in $F|_\alpha$ **then** — Add all asymmetric literals

    $\alpha := \alpha \cup \{u\}$ for each unit u

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      $G := \{D|_\alpha \mid (D \vee \neg l) \in F \text{ and } D|_\alpha \neq \top\}$

      if $G = \emptyset$ **then return** (**true**, $\sigma \cdot (\neg E_l : E)$) — Blocking Check on l

      $\Phi := \bigcap G$

      if $\Phi \neq \emptyset$ **then**

        $\sigma := \sigma \cdot (\neg E_l : E)$ — Record to witness seq.

        $E := E \cup \Phi$ — Add all literals covered by l

        $\alpha := \alpha \cup \neg\Phi$

until no updates to α

return (**false**, ε)

Witnesses for Covered Clauses

- Witness sequence can be quadratic in the size of C_{ext}

$$|\omega_n| + \cdots + |\omega_1|$$

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- C is covered: $(a \vee b) \rightarrow (a \vee b \vee x) \rightarrow (a \vee b \vee x \vee y) \rightarrow \top$

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- If $b \in \omega$ then ω satisfies G

Covered Clauses are not PR

Cor. Not all covered clauses are PR

Same set up as before:

$$C = (a \lor b)$$

$$F = (\neg a \lor x) \land (\neg b \lor y) \land (\neg x \lor \neg y) \land \mathcal{S}(G, x, y)$$

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- Take $G = (c \vee d) \wedge (\neg c \vee d) \wedge (c \vee \neg d) \wedge (\neg c \vee \neg d)$
- No ω for C satisfies $F|_\alpha \vdash_1 F|_\omega$

Deciding PR vs. R

- Deciding whether a clause is PR is NP-complete [10]

- Main goal: find useful PR clauses while solving
- SDCL [8] use **reducts** to detect PR clauses
  - Small propositional formulas (ideally easy to solve)
  - (Un-)satisfiable reduct $\Rightarrow$ C is (not) PR w.r.t. F

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- Can we detect **R** clauses the same way?

Complexity of R

Difference Polynomial Time

Difference Polynomial Time or **DP** [15] is the class:

$$\mathrm{DP} = \{L_1 \setminus L_2 \mid L_1, L_2 \in \mathrm{NP}\}$$

- Defined to classify various "exact" or "critical problems"
- Ex: can a graph G be colored using exactly four colors?

$$\mathrm{NP}, \text{co-NP} \subseteq \mathbf{DP} \subseteq \Theta_2^{\mathrm{P}} = \mathrm{P}^{\mathrm{NP}[\mathcal{O}(\log)]}$$

- Second-level of the **Boolean hierarchy** over NP [1]
- BH collapse $\Rightarrow$ PH collapse [11]

Complexity of R

Thm. Deciding R is co-DP-complete

- Proof sketch: show complement is complete for DP

$$(F, C) \in \overline{\mathrm{R}} \text{ if } C \text{ is not redundant w.r.t. } F$$

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$$(F, C) \in \overline{\mathrm{R}} \text{ if } C \text{ is not redundant w.r.t. } F$$

- $\overline{\mathrm{R}} = \{(F, C) \mid F \text{ is SAT but } F \wedge C \text{ is UNSAT}\}$
  $= \{(F, C) \mid F \in \mathrm{SAT}\} \setminus \{(F, C) \mid F \wedge C \in \mathrm{SAT}\}$
  $\in \mathrm{DP}$

Complexity of R

Thm. Deciding R is co-DP-complete

- Reduction from SAT-UNSAT: (F, G) s.t. F is SAT, G is UNSAT
- Given F and G, let $C' = x$ and construct:

$$F' = \bigwedge_{C \in F} (C \vee x) \bigwedge_{D \in G} (D \vee \neg x)$$

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- Given F and G, let $C' = x$ and construct:

$$F' = \bigwedge_{C \in F} (C \vee x) \bigwedge_{D \in G} (D \vee \neg x)$$

- C' is not redundant w.r.t. F' $\iff$ $(F, G) \in$ SAT-UNSAT
- Notice:

$$F'|_x = \bigwedge_{D \in G} (D) = G$$

$$F'|_{\neg x} = \bigwedge_{C \in F} (C) = F$$

Conclusion

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Thanks!

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